

H. Leeb and S. Wilmsen

Institut für Kernphysik, Technische Universität Wien, Wiedner Hauptstraße 8-10/142, A-1040 Vienna, Austria

(February 25, 2000)

An exact determination of the size of the pseudospin symmetry violating part of the nucleon-nucleus scattering amplitude from scattering observables is presented. The approximation recently used by Ginocchio turns out to underestimate the violation of pseudospin symmetry. Nevertheless the conclusion of a modestly broken pseudospin symmetry in proton- ^{208}Pb scattering at $E_L = 800$ MeV remains valid.

I. INTRODUCTION

The concept of pseudospin was originally introduced [1,2] to explain the quasidegeneracy of spherical shell orbitals with nonrelativistic quantum numbers $(n_r, \ell, j = \ell + \frac{1}{2})$ and $(n_r - 1, \ell + 2, j = \ell + \frac{3}{2})$, where n_r , ℓ and j are the single-nucleon radial, orbital angular momentum and total angular momentum quantum numbers. This symmetry holds approximately also for deformed nuclei [3–5] and even for the case of triaxiality [6,7]. Only recently, Ginocchio and coworkers [8–11] pointed out that the origin of the approximate pseudospin symmetry is an invariance of the Dirac Hamiltonian with $V_V = -V_S$ under specific $SU(2)$ transformations [12]. Here, V_S and V_V are the scalar and vector potentials, respectively. In the non-relativistic limit this leads to a Hamiltonian which conserves pseudospin,

$$\tilde{s} = 2 \frac{\mathbf{s} \cdot \mathbf{p}}{p^2} \mathbf{p} - \mathbf{s}, \quad (1)$$

where \mathbf{s} is the spin and \mathbf{p} is the momentum operator of the nucleon. For realistic mean fields there must be at least a weak pseudospin-symmetry violation because otherwise no bound state exists [8].

Already in 1988 Bowlin et al. [13] investigated whether the symmetry associated with $V_V = -V_S$ manifests itself also in proton-nucleus scattering. They evaluated the analyzing power $P(\theta)$ and the spin rotation function $Q(\theta)$ under the assumption $V_V = -V_S$, where θ is the scattering angle. Since the experimental data deviate significantly from their prediction they concluded that the symmetry is destroyed for low-energy proton scattering; only at high energies some remnants might survive. However, recently, Ginocchio [14] revisited this question within the scattering formalism in terms of pseudospin. Based on a first-order approximation he extracted from experimental proton- ^{208}Pb scattering data at $E_L = 800$ MeV [15] the pseudospin-symmetry breaking part of the scattering amplitude. Contrary to [13] he obtained at all angles a relatively small pseudospin dependent part of the scattering amplitude which confirms the relevance of pseudospin symmetry also for proton-nucleus scattering at least at medium energies.

In the present work we reexamine the question of pseudospin symmetry in nucleon-nucleus scattering and derive an exact relation for the nucleon-nucleus scattering amplitude in terms of scattering observables. The exact relationship for the pseudospin symmetry violating part of the scattering amplitude differs in an essential way from the first-order expression used by Ginocchio [14]. Using the same proton- ^{208}Pb scattering data at 800 MeV the exact relationship leads at all measured angles to a significantly increased violation of the pseudospin symmetry as compared to [14]. Nevertheless, the size of the violation remains within the limits which allow one to consider pseudospin symmetry as a relevant symmetry in nucleon-nucleus scattering.

In section II we discuss briefly the basic relations between the scattering observables and the scattering amplitude of nucleon-nucleus scattering within the standard formalism. Introducing the proper transformation to a pseudospin representation we present in section III exact relations for the pseudospin-symmetry violating part of the scattering amplitude. A first application of the exact relations is given in section IV where we consider the example of proton- ^{208}Pb scattering data at $E_L = 800$ MeV. Concluding remarks are given in section V.

II. SPIN DEPENDENT SCATTERING AMPLITUDE

The scattering amplitude, f , for the elastic scattering of a nucleon with momentum k on a spin zero target is given by [16],

$$f = A(k, \theta) + B(k, \theta) \boldsymbol{\sigma} \cdot \hat{\mathbf{n}}. \quad (2)$$

Here σ is the vector formed by the Pauli matrices, \hat{n} is the unit vector perpendicular to the scattering plane and θ is the scattering angle. The complex-valued functions $A(k, \theta)$ and $B(k, \theta)$ are the spin-independent and the spin-dependent parts of the scattering amplitude. Both are not fully accessible to measurement. The observables in nucleon-nucleus scattering are related to intensities and can be described in terms of the amplitudes $A(k, \theta)$ and $B(k, \theta)$. In the case of the scattering of a spin- $\frac{1}{2}$ particle by a spinless target the observables are the differential cross section

$$\frac{d\sigma}{d\Omega}(k, \theta) = |A(k, \theta)|^2 + |B(k, \theta)|^2, \quad (3)$$

the polarization

$$P(k, \theta) = \frac{B(k, \theta)A^*(k, \theta) + B^*(k, \theta)A(k, \theta)}{|A(k, \theta)|^2 + |B(k, \theta)|^2}, \quad (4)$$

and the spin rotation function

$$Q(k, \theta) = i \frac{B(k, \theta)A^*(k, \theta) - B^*(k, \theta)A(k, \theta)}{|A(k, \theta)|^2 + |B(k, \theta)|^2}. \quad (5)$$

From Eqs. (4) and (5) follows $P^2 + Q^2 \leq 1$.

The extraction of the full scattering amplitude (moduli and phases of $A(k, \theta)$ and $B(k, \theta)$) from measurements is a very challenging question in quantum mechanics and intimately related to the longstanding *phase problem* in structure physics. Here, only a partial solution of this problem is required because our interest is limited to the ratio $|R_s(k, \theta)| = |B(k, \theta)|/|A(k, \theta)|$ which is a measure of the strength of the spin-dependent interaction. Combining Eqs. (4) and (5) provides a phase relation between the amplitudes,

$$\sqrt{\frac{P - iQ}{P + iQ}} = \exp i(\phi_B - \phi_A), \quad (6)$$

where ϕ_A and ϕ_B are the phases of the amplitudes A and B , respectively. Using this result either in Eq. (4) or (5) leads to a quadratic equation for the ratio of the moduli,

$$1 - \frac{2}{\sqrt{P^2 + Q^2}}|R_s| + |R_s|^2 = 0, \quad (7)$$

which has two solutions. Using the condition that $P = Q = 0$ implies $|B| = 0$ one can immediately select the physical solution ($|A| \neq 0$),

$$|R_s| = \frac{|B|}{|A|} = \frac{1 - \sqrt{1 - P^2 - Q^2}}{\sqrt{P^2 + Q^2}} = \frac{\sqrt{P^2 + Q^2}}{1 + \sqrt{1 - P^2 - Q^2}}. \quad (8)$$

This result together with Eq. (6) yields also the ratio of the amplitudes,

$$R_s = \frac{B}{A} = \frac{P - iQ}{1 + \sqrt{1 - P^2 - Q^2}}. \quad (9)$$

These relations are exact at all scattering angles and energies.

For comparison with the recent work of Ginocchio [14] we evaluate $|R_s|^2$ from Eq. (9),

$$|R_s|^2 = C_s \left[\left(\frac{P}{2} \right)^2 + \left(\frac{Q}{2} \right)^2 \right] \quad (10)$$

with

$$C_s = \frac{4}{2 + 2\sqrt{1 - P^2 - Q^2} - P^2 - Q^2} \geq 1. \quad (11)$$

The second factor (square brackets) on the right-hand side of Eq. (10) is the first-order expression used in [14]. It is obvious from the correction factor C_s that the first-order approximation systematically underestimates the quantity $|R_s|^2$.

Eqs. (6) and (9) represent important pieces for the proper determination of the scattering amplitude from nucleon-nucleus scattering observables. In addition one can derive from Eq. (3) with the use of Eq. (9) the relationship,

$$|A|^2 = \frac{d\sigma}{d\Omega} \left(1 - \frac{P^2 + Q^2}{2 + 2\sqrt{1 - P^2 - Q^2}} \right). \quad (12)$$

Combining Eqs. (6), (9) and (12) one can determine the full scattering amplitude up to an overall phase. Thus we have reduced the determination of the scattering amplitude in a system with spin $\frac{1}{2}$ to a problem similar to that with spin zero.

III. VIOLATION OF PSEUDOSPIN SYMMETRY

The closed-form expression (9) for the spin-dependent term indicates the possibility of deriving an exact expression for the violation of the pseudospin symmetry. For this purpose the concept of pseudospin must be introduced in the formalism of nucleon-nucleus scattering. Specifically, the partial-wave expansion of the scattering amplitude must be performed in terms of the pseudo-orbital angular momentum,

$$\begin{aligned} \tilde{\ell} &= \ell + 1, \text{ for } j = \ell + 1/2, \\ \tilde{\ell} &= \ell - 1, \text{ for } j = \ell - 1/2. \end{aligned} \quad (13)$$

Accordingly, the partial-wave S-matrix elements $S_{\ell,j}$ must be defined for pseudo-orbital angular momentum ($\tilde{S}_{\ell,j}$) as

$$\tilde{S}_{\tilde{\ell},j=\tilde{\ell}-1/2} = S_{\tilde{\ell}-1,j=\tilde{\ell}-1/2}, \quad \tilde{S}_{\tilde{\ell},j=\tilde{\ell}+1/2} = S_{\tilde{\ell}+1,j=\tilde{\ell}+1/2}. \quad (14)$$

As shown by Ginocchio [14] the scattering amplitudes \tilde{A} and \tilde{B} are related to A and B by a unitary transformation

$$\begin{pmatrix} \tilde{A} \\ \tilde{B} \end{pmatrix} = \begin{pmatrix} \cos(\theta) & i \sin(\theta) \\ i \sin(\theta) & \cos(\theta) \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix}. \quad (15)$$

Using the transformed amplitudes the scattering observables are then given by

$$\frac{d\sigma}{d\Omega} = |\tilde{A}|^2 + |\tilde{B}|^2, \quad (16)$$

$$P = \frac{\tilde{B}\tilde{A}^* + \tilde{B}^*\tilde{A}}{|\tilde{A}|^2 + |\tilde{B}|^2}, \quad (17)$$

$$Q = \frac{\sin(2\theta) \left[|\tilde{A}|^2 - |\tilde{B}|^2 \right] + i \cos(2\theta) \left[\tilde{B}\tilde{A}^* - \tilde{B}^*\tilde{A} \right]}{|\tilde{A}|^2 + |\tilde{B}|^2}. \quad (18)$$

In the limit of pseudospin symmetry the amplitude \tilde{B} vanishes and consequently $P = 0$ and $Q = \sin(2\theta)$ [13]. Therefore the ratio $R_{ps} = \tilde{B}/\tilde{A}$ is a measure of the violation of pseudospin symmetry in nucleon-nucleus scattering. With the transformation (15) it is straightforward to express R_{ps} in terms of R_s ,

$$R_{ps} = \frac{\tilde{B}}{\tilde{A}} = \frac{i \tan(\theta) + R_s}{1 + i \tan(\theta) R_s} \quad (19)$$

and

$$|R_{ps}|^2 = \frac{\tan^2(\theta) - Q \tan(\theta) + |R_s|^2(1 - Q \tan(\theta))}{1 + Q \tan(\theta) + |R_s|^2(\tan^2(\theta) + Q \tan(\theta))}. \quad (20)$$

Substitution of Eq. (9) in Eq. (19) yields an exact relation for the violation of pseudospin symmetry in terms of scattering observables,

$$R_{ps} = \frac{P + i \left[\left(1 + \sqrt{1 - P^2 - Q^2} \right) \tan(\theta) - Q \right]}{\left[\left(1 + \sqrt{1 - P^2 - Q^2} \right) + Q \tan(\theta) \right] + iP \tan(\theta)}. \quad (21)$$

For comparison with the recent work of Ginocchio [14] we evaluate $|R_{ps}|^2$ from Eq. (18). By straightforward algebraic manipulations one obtains

$$|R_{ps}|^2 = C_{ps} \left[\left(\frac{P}{2} \right)^2 + \left(\frac{Q - \sin(2\theta)}{2 \cos(2\theta)} \right)^2 \right] \quad (22)$$

with

$$C_{ps} = \frac{(1 + |R_{ps}|^2)^2}{1 + |R_{ps}|^2 \tan^2(2\theta) + i (R_{ps}^* - R_{ps}) \tan(2\theta)}. \quad (23)$$

The factor in square brackets on the right-hand side of Eq. (22) corresponds to the first-order expression used in [14] while C_{ps} is a correction factor. Eqs. (22) and (23) are not the best suited for the evaluation of $|R_{ps}|^2$, however, they demonstrate clearly that the actual value of the pseudospin symmetry breaking amplitude may deviate significantly from the first-order approximation.

IV. EXAMPLE OF PROTON-NUCLEUS SCATTERING

The relations derived in section II and III can be directly used for an analysis of proton-nucleus scattering data, where accurate measurements of the analyzing power $P(\theta)$ and the spin rotation function $Q(\theta)$ are available. Specifically, we consider analyzing power [17] and spin rotation function measurements [15] for proton- ^{208}Pb scattering at $E_L = 800$ MeV and evaluate the ratios $|R_s|^2$ and $|R_{ps}|^2$.

The violation of pseudospin symmetry can be read off from Fig. 1, where the ratio $|R_{ps}|^2$ is displayed as a function of the scattering angle. In Fig. 1 the ratios are shown only at those scattering angles where measured values of the spin rotation function [15] are available. The corresponding values of the analyzing power have been obtained by linear interpolation of the more complete data set given by [17]. The shown uncertainties result from a linear error propagation of the given experimental uncertainties in $P(\theta)$ and $Q(\theta)$. For comparison the results of [14] are also displayed. It is obvious from Fig. 1 that the first-order approximation underestimates the pseudospin symmetry breaking part of the scattering amplitude at all scattering angles, particularly at the highest available ones. Furthermore it is interesting to note that the uncertainties obtained with the use of the exact relations are consistently larger than those obtained in first-order approximation.

In Fig. 2 the corresponding results are shown for the ratio $|R_s|^2$ characterizing the spin dependent part of the scattering amplitude. As already seen from the exact formulation (10) the first-order approximation underestimates systematically the spin dependent part. The difference depends only on the absolute size of $P^2 + Q^2$ and amounts to a factor 2 at $\theta = 15$ degrees.

The application of Eqs. (6), (9) and (12) to scattering data is straightforward and yields $|A(k, \theta)|$, $|B(k, \theta)|$ and $\phi_B(k, \theta) - \phi_A(k, \theta)$. Up to a common phase, therefore, the scattering amplitude can be extracted from the scattering observables.

V. CONCLUSIONS

We have derived closed-form expressions for the relationship between scattering amplitude and observables in nucleon-nucleus scattering. The scattering amplitudes can be determined, up to a common phase factor, from measured differential cross section, analyzing power and spin rotation data. Hence the problem of extracting the scattering amplitudes from nucleon-nucleus scattering data becomes mathematically similar to that for spinless particles.

Extending these relations we derived a closed-form expression for the ratio of the pseudospin dependent and independent amplitudes. Thus we obtain an exact measure of the violation of pseudospin symmetry. Comparison with the first-order expression of [14] exhibits significant deviations. Using experimental proton- ^{208}Pb data at $E_L = 800$ MeV of [15,17] we find an increased violation of pseudospin symmetry in the whole available angular range as compared to the results of [14]. The increment is up to 40% so that $|R_{ps}|^2$ reaches values of about 0.15 at $\theta = 15$ degrees. Nevertheless it remains within the limits which allow one to consider the pseudospin symmetry to be a relevant concept in nucleon-nucleus scattering at intermediate energies. At lower energies increased violation of pseudospin symmetry may occur [13]. Whether this is true has to be proved by studying the energy dependence of $|R_{ps}|^2$ for several nuclei.

The authors thank Prof. Dr. G.W. Hoffmann and Prof. Dr. L. Ray for making available the experimental data in tabular form and Prof. Dr. R. Lipperheide for fruitful discussions and a careful reading of the manuscript.

- [1] K.T. Hecht and A. Adler, Nucl. Phys. **A137**, 129 (1969).
- [2] A. Arima, M. Harvey, and K. Shimizu, Phys. Lett **30B**, 517 (1969).
- [3] R.D. Ratna Raju, J.P. Draayer, and K.T. Hecht, Nucl. Phys. **A202**, 433 (1973).
- [4] J.P. Draayer and K.J. Weeks, Ann. Phys. (N.Y.) **156**, 41 (1984).
- [5] J.Y. Zeng, J. Meng, C.S. Wu, E.G. Zhao, Z. Xing, and X.Q. Chen, Phys. Rev. C **44**, R1745 (1991).
- [6] A.L. Blokhin, T. Beuschel, J.P. Draayer, and C. Bahri, Nucl. Phys. **A612**, 163 (1997).
- [7] T. Beuschel, A.L. Blokhin, and J.P. Draayer, Nucl. Phys. **A619**, 119 (1997).
- [8] J.N. Ginocchio, Phys. Rev. Lett. **78**, 436 (1997).
- [9] J.N. Ginocchio and D.G. Madland, Phys. Rev. C **57**, 1167 (1998).
- [10] J.N. Ginocchio and A. Leviatan, Phys. Lett. B **425**, 1 (1998).
- [11] J.N. Ginocchio, Phys. Rep. **315**, 231 (1999).
- [12] J.S. Bell and H. Ruegg, Nucl. Phys. **B98**, 151 (1975).
- [13] J.B. Bowlin, A.S. Goldhaber, and C. Wilkin, Z. Phys. A **331**, 83 (1988).
- [14] J.N. Ginocchio, Phys. Rev. Lett. **82**, 4599 (1999).
- [15] R.W. Fergerson, M.L. Barlett, G.W. Hoffmann, J.A. Marshall, E.C. Milner, G. Pauletta, L. Ray, J.F. Amann, K.W. Jones, J.B. McClelland, M. Gazzaly, and G.J. Igo, Phys. Rev. C **33**, 239 (1986).
- [16] H. Feshbach, *Theoretical Nuclear Physics – Nuclear Reactions* (John Wiley and Sons, Inc., New York, 1992).
- [17] G.W. Hoffmann, G.S. Blanpied, W.R. Coker, R.P. Liljestrang, N.M. Hintz, M.A. Oothoudt, T.S. Bauer, G. Igo, G. Pauletta, C.A. Whitten, Jr., D. Madland, J.C. Pratt, L. Ray, J.E. Spencer, H.A. Thiessen, H. Nann, K.K. Seth, C. Glashauser, D.K. McDaniels, J. Tiensley, and P. Varghese, Phys. Rev. Lett **40**, 1256 (1978).

Figure 1

The angular dependence of the ratio $|R_{ps}|^2$ evaluated from proton- ^{208}Pb scattering data at $E_L = 800$ MeV. The solid error bars are obtained by the exact formulation (20), the dashed error bars are the results of the first-order approximation. The lines connecting the datapoints are drawn as a guidance for the eye.

Figure 2

The angular dependence of the ratio $|R_s|^2$ evaluated from proton- ^{208}Pb scattering data at $E_L = 800$ MeV. The solid error bars are obtained by the exact formulation (20), the dashed error bars are the results of the first-order approximation. The lines connecting the datapoints are drawn as a guidance for the eye.

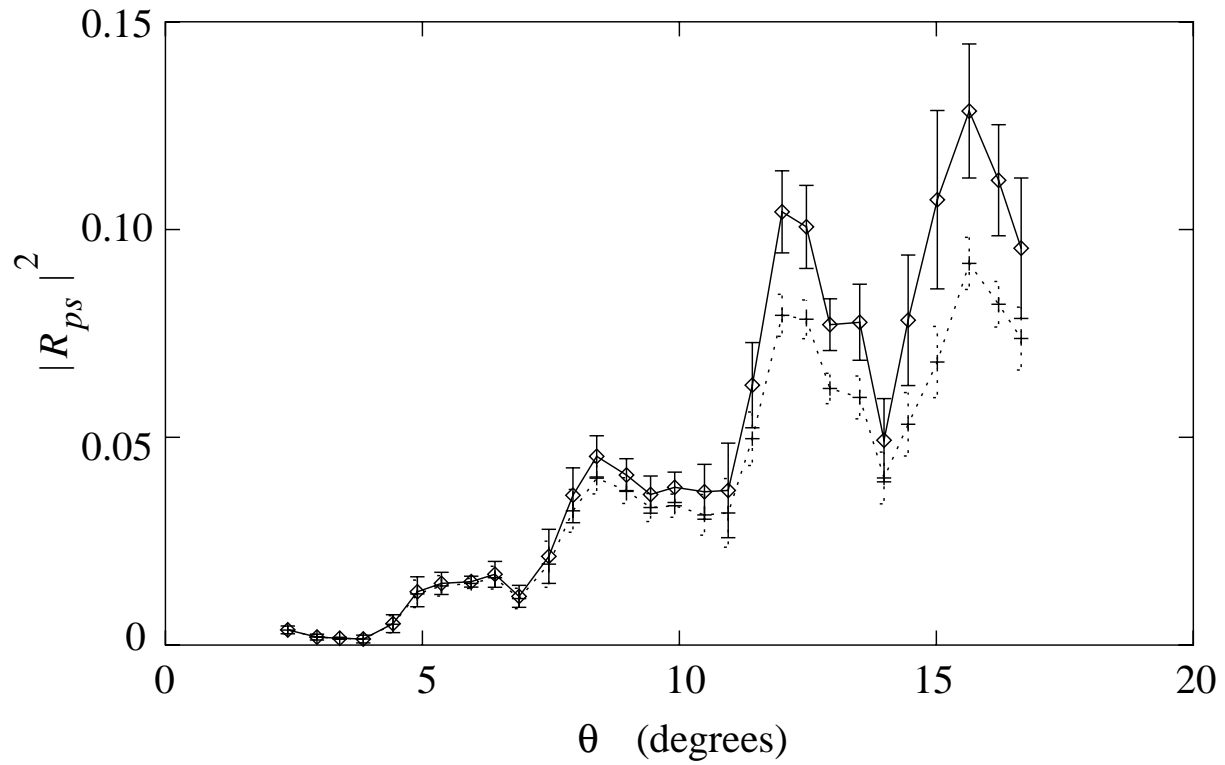


Figure 1

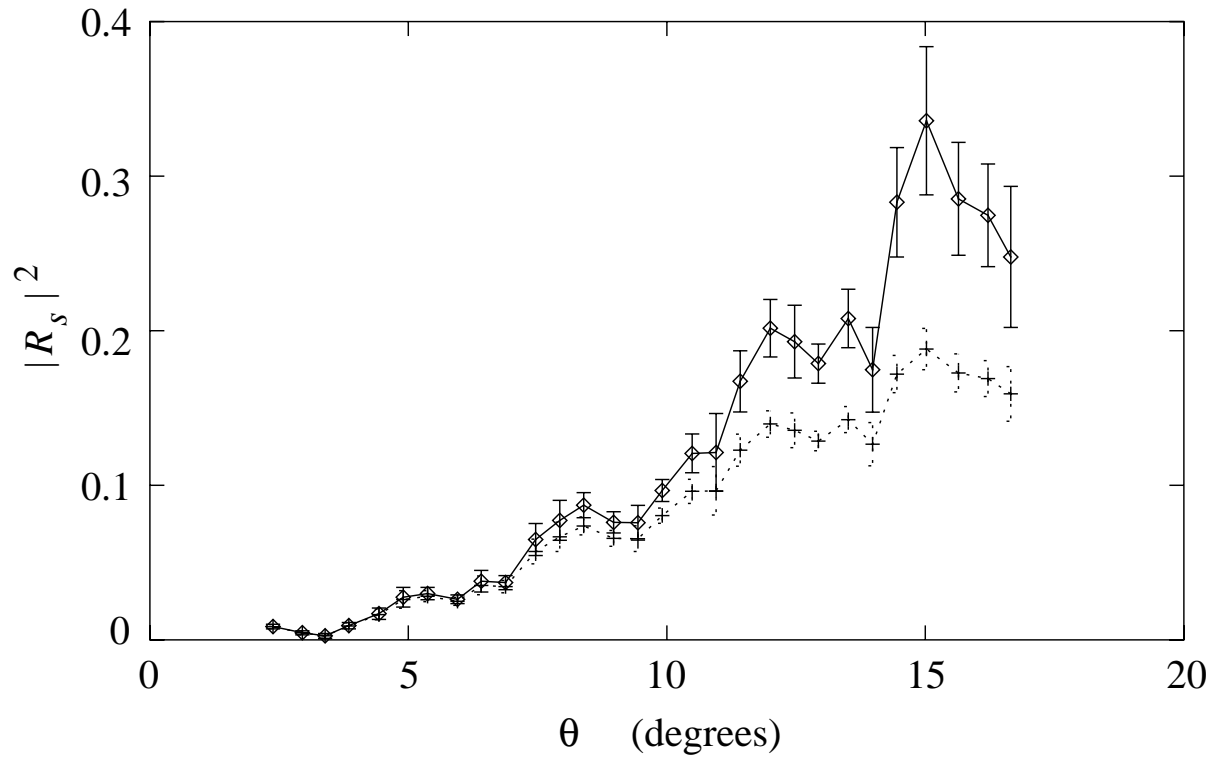


Figure 2